Advances in the micromorphic homogenization of dynamical metamaterials

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The engineering design and analysis of high-tech systems calls for reliable and accurate methods that adequately capture the mechanics of the underlying constituting materials. This tends to be particularly complex for highly heterogeneous materials, for which a rich class of homogenization methods has been developed to condense all the small-scale fluctuations in an effective continuum that can be solved at the engineering level. Within this class, computational homogenization serves as a highly accurate two-scale coupling of complex nonlinear materials. Amongst others, this homogenization method has been successfully applied to mechanical, thermal and electromagnetic problems, but its application to metamaterials remains a challenge. Metamaterials are characterized by tailored microstructures that entail a dominant emergent effect at the macro-scale. This contribution focuses on the advanced homogenization and model reduction of dynamical metamaterials, leading to a micromorphic-like continuum.

Dynamical metamaterials, mitigating waves, are instrumental for inhibiting sound and vibration transmission in a targeted frequency range. As a point of departure, the original computational homogenization scheme applicable to elastic resonant acoustic metamaterials will be outlined [1]. Exploiting linearity of the problem, a closed form micromorphic continuum homogenization approach for this class of materials is obtained. The resulting dispersion spectra are shown to be accurately captured, which enables direct solutions of initial boundary value problems as required for engineering design problems at the macro-scale [2]. Next, the method will be extended to viscoelastic metamaterials, whereby the damping effects in the solid are modeled using the Kelvin-Voigt constitutive behavior [3, 4]. Finally, a solid-fluid metamaterial is considered, for which an elastic Biot continuum is recovered at the macroscale with micromorphic enrichment variables representing the inertia of the local structure resonance.

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