

Modelling electrical conductivity across scales

Andreas Menzel^{1,2}

¹Institute of Mechanics, TU Dortmund University, Dortmund, Germany
e-mail: andreas.menzel@udo.edu

²Division of Solid Mechanics, Lund University, Lund, Sweden
e-mail: andreas.menzel@solid.lth.se

KEYWORDS: *homogenisation, thermo-electric conductors, Joule heating, cohesive-zone model, size effects.*

Motivated by the effects induced by grain boundaries and microcracks on effective electrical properties, the focus of this contribution is on the development of generalised computational multiscale formulations for electrical conductors. In line with classic energy-based approaches for bulk material, it is shown that effective electrical conductivity tensors can be condensed from the underlying microstructure. Thereby, special emphasis is placed on asymptotic expansions and Hill-Mandel-type multiscale techniques. Their usefulness is demonstrated by consideration of experimental investigations. Extending the established multiscale description of electrical conductors, grain boundaries together with related strong discontinuities in the microscale fields are elaborated. This allows for a comprehensive analysis of the Andrews method, which is established as the key materials science approach to study grain boundary resistivity and its effect on the effective electrical properties of polycrystalline materials. Particularly, the applicability as well as the limitations of the Andrews method are addressed. In general, material interfaces occur at different length scales – electrically conductive adhesives being a typical example of, e.g., macroscale interfaces. Such adhesives are key elements of electronic packages used in, e.g., communication and computing applications, with their distinct properties induced by the underlying multiscale nature. Considering appropriate scale-bridging relations, these macroscale composite interphases are approximated as zero-thickness cohesive interfaces. The proposed framework thereby generalises classic phenomenological traction-separation approaches by relating the apparent electro-mechanical interface response to the underlying microstructure.

ACKNOWLEDGEMENT: Financial support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Project-ID 278868966 – TRR 188 is gratefully acknowledged.

- [1] D. Güzel, D. Wiedemann, T. Kaiser, and A. Menzel. On dissipative effects in thermo-electrically coupled systems: Hill-Mandel-type homogenisation, asymptotic expansions and two-scale convergence. *J. Mech. Phys. Solids*, 208:106427, 2026. doi:10.1016/j.jmps.2025.106427.
- [2] D. Güzel, T. Kaiser, and A. Menzel. A computational multiscale approach towards the modelling of microstructures with material interfaces in electrical conductors. *Math. Mech. Solids*, 30(2):247–266, 2025. doi:10.1177/10812865231202721.
- [3] D. Güzel, T. Kaiser, H. Bishara, G. Dehm, and A. Menzel. Revisiting Andrews method and grain boundary resistivity from a computational multiscale perspective. *Mech. Mat.*, 198:105115, 2024. doi:10.1016/j.mechmat.2024.105115.
- [4] T. Kaiser, G. Dehm, C. Kirchlechner, A. Menzel, and H. Bishara. Probing porosity in metals by electrical conductivity: Nanoscale experiments and multiscale simulations. *Euro. J. Mech. A/Solids*, 97:104777, 2023. doi:10.1016/j.euromechsol.2022.104777.
- [5] T. Kaiser and A. Menzel. Fundamentals of electro-mechanically coupled cohesive zone formulations for electrical conductors. *Comput. Mech.*, 68:51–67, 2021. doi:10.1007/s00466-021-02019-z.