Potential method in the steady vibration problems for the coupled theory of Moore-Gibson-Thompson thermoporoelasticity

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KEYWORDS: *Thermoporoelasticity, Moore-Gibson-Thompson Law, Steady Vibration, Potential Method.*

The first theory of poroelasticity, based on Darcy's law, was formulated by Biot (1941), who developed an elasticity framework describing the mechanical behavior of fluid-saturated porous solids. Moreover, the linear theory of poroelasticity, based on the volume fraction concept was first presented by Cowin and Nunziato (1983). Evidently, theories based on Darcy's law account for variation of the pore fluid pressure but neglect change of the pore volume fraction. Conversely, theories grounded on the concept of volume fraction incorporate change of pore volume fraction while disregarding variation of fluid pressure. Both approaches, however, capture only part of the physical picture. In reality, deformation of a porous body inevitably induces simultaneous changes in both the pore volume fraction and the pore fluid pressure. This limitation highlights the necessity of coupled models that treat these effects in a unified framework.

To overcome this limitation, Svanadze (2020) developed a coupled theory of poroelasticity, in which both physical effects are incorporated within a unified framework. This coupled approach generalizes Biot’s and Cowin-Nunziato’s theories by introducing additional field variables and interaction terms. Based on Fourier’s classical law of heat conduction, Svanadze (2019) introduced a coupled linear theory of thermoporoelasticity.

In this work, based on the Moore-Gibson-Thompson (MGT) law of heat conduction, the coupled linear theory of thermoporoelasticity is proposed in which the combination of the following four mechanical phenomena is considered: the deformation of elastic porous material, Darcy's law for fluid flow in pore network, the concept of the volume fraction of pores, and the MGT law of heat conduction. On the basis of this theory the basic boundary value problems (BVP) of steady vibrations are investigated using the potential method (the boundary integral equations method). Namely, the fundamental solution of the system of steady vibration equations is constructed explicity by elementary functions. Green’s identities are obtained and the uniqueness theorems for the classical solutions of the BVPs are proved. Some useful singular integral operators are introduced and the basic properties of these operators are presented. Then, the above-mentioned BVPs of steady vibrations are reduced to the equivalent always solvable singular integral equations for which Noether's theorems are valid. Finally, the existence theorems for classical solutions of the BVPs of steady vibrations are proved by means of the potential method and the theory of singular integral equations.

ACKNOWLEDGEMENT: This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [Grant # FR-23-4905]