Surface energetic elasticity problems for fractures and thin inclusions

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It is well-known that surface energy plays an increasingly important role in modeling of objects at nano-scales. In this talk, surface energy model proposed by Steigmann and Ogden will be applied to fracture problems and to problems with thin inclusions in two-dimensional setting. Introduction of surface energy leads to non-classical boundary conditions incorporating both stresses and derivatives of displacements. An importance of tip conditions, which is often overlooked in literature, will be explored. The mechanical problem will be reduced to systems of singular integro-differential equations which are further reduced to systems of weakly-singular equations. The existence and uniqueness of the solutions will be discussed. The effects of introduction of Surface mechanics on the solution fields willbe investigated, including stress and strain singularities at the tips of inclusions and cracks, and size-dependency in the system. Both of those represent significant divergence of the studied theories from classic results. A numerical schemes of the solution of the relevant systems of integral and integro-differential equations will be suggested. The numerical results, parametric studies, and comparisons with known in the literatura results will be presented.

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