On regularization of the Isotropic Material Design

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The paper concerns the d-dimensional problem of minimization of the compliance of linear elastic structures made of an isotropic material; d=2 or d=3. The bulk k and shear μ moduli are the design variables, both viewed as non-negative fields on the design domain Ω. The design variables are subject to the isoperimetric condition which is the upper bound of L^p-norm of the elastic moduli:

$$\left(\int_\Omega \left[(dk)^{1+e} + \left(\frac{1}{2}d(d+1)-1\right)(2\mu)^{1+e}\right] dx\right)^{\frac{1}{1+e}} \le \Lambda$$

The case of e=0 corresponds to the original concept of the *Isotropic Material Design* (*IMD*) method proposed in the paper by Czarnecki [1]. Then the unit cost represents the trace of the Hooke tensor. The original IMD method leads to the variational problems with integrands of linear growth, hence its correct setting requires the measure-theoretic tools, see [2]. Thus, it is highly recommended to regularize the IMD. In the present paper the IMD method will be extended by assuming the above L^p -norm-based cost condition. At the first step the design variables are eliminated. Thus, the optimum design problem is reduced to the pair of mutually dual problems of the mathematical structure of a theory of elasticity of an isotropic body with nonlinear power-law type constitutive equations. They link the trace of the stress with the trace of the strain and, independently, the deviator of the stress with the deviator of the strain. Both the states of stress and strain determine the optimal layouts of the bulk and shear moduli of the least compliant structure, see [3]. The effective non-linear static problem can be solved by using available commercial codes, e.g. with the help of Abaqus. This paves the way towards solving topology optimization problems on a large scale.

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