A Universal Laplace Transform–Based Algorithm for the Analysis of Statically Indeterminate Bending Beams Including Shear Deformation Effects in the Vlasov–Timoshenko Model

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This contribution outlines a unified analytical framework based on a Laplace-transform approach for the static analysis of prismatic bending beams across multiple classical and extended beam models. Starting from the full Timoshenko theory [1], the governing equation is systematically reduced to the Euler–Bernoulli model [2] as a limiting case of vanishing shear deformation. The formulation also incorporates a Vlasov warping correction for asymmetric cross-sections, enabling the analysis of beams lacking geometric symmetry and exhibiting warping-related stiffness effects.

The study addresses various static load types, including concentrated forces, bending moments, distributed loads, and beams reinforced with two internal stiffeners. All computations are performed in the Laplace domain using a single governing equation, even for multi-span or piecewise-loaded beams, and inverse transformation yields closed-form deflection fields.

To assess the influence of support spacing on the agreement between the Timoshenko, Euler–Bernoulli and Vlasov-corrected models, an experimental campaign was conducted in which beam deflections were measured at several points and compared with analytical predictions for varying support configurations. The results identify the ranges of beam slenderness and support geometry for which shear deformation, warping stiffness, or classical bending assumptions dominate.

The main novelty lies in solving multi-span and irregularly supported beams using a single closed-form Laplace-space formulation. The algorithm includes (i) system release from supports, (ii) transformation of loading to the Laplace domain, (iii) analytical solution in Laplace space, (iv) inverse transformation to the physical domain, (v) enforcement of boundary and intermediate support conditions, and (vi) computation of deflections, slopes, bending moments, shear forces, and reactions, enabling efficient treatment of beams with multiple supports, non-classical stiffness features, and non-uniform loading patterns.

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