Some Results and Unusual Consequences of Elasticity without Hooke's law

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Hookean elasticity is based on a linear relation between stresses and strains, which is reasonable for the most of solids at small deformations. In turn, linearity justifies the superposition principle leading to the classical generalized Hooke law. One of its well-known thermodynamical consequences is that for isotropic materials Poisson's ratio lies within the interval $[-1, 0.5]$. In this contribution, we propose a hyperelastic isotropic material model based on a positive-definite strain energy function (see [1] for more details). The stress-strain response of the model is non-linear even at infinitesimal deformations and cannot thus be linearized. As a result, the superposition principle does not apply, and the generalized Hooke law is not valid even at small strains. In this case, one can speak about non-Hookean infinitesimal elasticity. A further unusual feature of this material model is that Poisson's ratio can be smaller than -1 or greater than 0.5 in full agreement with the laws of thermodynamics. For example, in the latter case volume of a specimen monotonically decreases under uniaxial tension, which is rather unusual. The proposed model is verified in various deformations states such as uniaxial tension/compression, simple shear and pure dilation and demonstrates plausible and stable response.

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