Random Static Analysis of Plates Resting on Elastic Foundations Considering One-sides Constraints by the Finite Element, Boundary Element and Finite Difference Methods

Marcin Kamiński[1], Michał Guminiak[2] and Agnieszka Lenartowicz[2,*]

[1]Faculty of Civil Engineering, Architecture and Environmental Engineering, Lodz University of Technology, Lodz, Poland, e-mail: marcin.kaminski@p.lodz.pl

[2]Faculty of Civil and Transport Engineering, Poznan University of Technology, Poznan, Poland
e-mails: michal.guminiak@put.poznan.pl, agnieszka.lenartowicz@put.poznan.pl

KEYWORDS: *Finite Element Method, Boundary Element Method, Finite Difference Method, Thin Plates, Elastic Foundations, Semi-Analytical Probabilistic Analysis, Stochastic Perturbation Technique, Monte Carlo Simulations.*

The paper presents a random analysis of the statics of a thin plate resting on an elastic foundation of the Winkler type and an elastic half-space. For the foundation of the Winkler type the one-sided constraints are applied too. The problem of bending the plate is independently described by the Finite Element Method (FEM), Boundary Element Method (BEM) and the Finite Difference Method (FDM). For the FEM approach, the plate is discretized using classical four-node rectangular finite elements with three degrees of freedom at each node [1]. In terms of the BEM two boundary and domain integral equations, with the boundary conditions formulated in such a way that it is not necessary to take into account the Kirchhoff forces [2]. Within the plate domain, there are introduced additional collocation points at which the elastic discrete support is realized and the fundamental solution for statics of a pure infinite plate is used. The plate boundary is divided into elements of the constant type. A non-singular approach is used to calculate boundary integrals. This approach allows for a significant simplification of the calculation algorithm. Finally, the formulation of the problem by the FDM allows to easily take into account the variable stiffness of the plate. A random analysis for selected design parameters is presented such as the plate thickness, its material properties, and the properties of the parameters describing the foundation itself. Three approaches were adopted: semi-analytical (SAM), stochastic perturbation technique (SPT) and Monte Carlo simulation (MCs), which can be traced, e.g., in [3, 4, 5]. For these three methods, random moments are defined, the most important of which is the expected value of the solution, e.g. the plate deflection established at selected point, as a function of the coefficient of variation of the selected design parameter. A Gaussian probability density distribution is assumed for the arbitrary design parameter. Since such a distribution is continuous, it is necessary to determine the so-called continuous response function for a finite number of deterministic solutions. Most often, these are 3rd or 4th order polynomials. Higher order polynomials provide some power "reserve" when differentiating them, which is necessary to determine higher order random moments as skewness or kurtosis.

[1] M. Kuczma, *Foundations of structural mechanics with shape memory. Numerical modeling*, University of Zielona Góra Publishing House, Zielona Góra, 2010, ISBN 978-83-7481-411-9 (in Polish).

[2] M. Guminiak, P. Litewka, R. Sygulski, *Static analysis of plates on the unilateral foundation by BEM*, Proc. of the 9th Int. Conf. "Modern building materials, structures and techniques", Vilnius, May 19–21, 2004, selected papers, Vilnius Gediminas Technical University Press "Technika" scientific book 1026, III, 765–768, 2004, eds.: E. K. Zavadskas, P. Vainiunas and F. M. Mazzolani.

[3] G. Stefanou, *The stochastic finite element method: Past, present and future*, Computer Methods in Applied Mechanics and Engineering, 198 (9–12), 2009, 1031–1051.

[4] M. Kamiński, *The Stochastic Perturbation Method for Computational Mechanics*, Wiley: Chichester, 2013.

[5] M. Kamiński, M. Przychodzki, M. Łasecka-Plura, M. Guminiak, A. Lenartowicz, *Random eigenvibrations of beams with viscoelastic layers*, Journal of Theoretical and Applied Mechanics 62(4), 2024, 763–767.