

## Stochastic Analysis of Deflection Response of RC Beam Strengthened with CFRP Materials

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**KEYWORDS:** RC beams, CFRP materials, carbon strips, Finite Element Method, Semi-Analytical Probabilistic Analysis, Stochastic Perturbation Technique, Monte Carlo Simulations.

This paper presents an analysis of the deflection state of the Reinforced Concrete (RC) simply supported beam strengthened with Carbon Fiber Reinforced Polymer (CFRP) [1, 2] strips in a random approach. In the deterministic analysis, the Finite Element Method (FEM) with 2D elements and the Concrete Damage Plasticity (CDP) model for concrete [3], taking into account cracking phenomena, are used. Random analysis is based on FEM [4]. In the current work three independent approaches were adopted: Semi-Analytical Method (SAM), Stochastic Perturbation Technique (SPT), and Monte Carlo simulation (MCS). These three approaches have already been used in previous works, e.g. [5, 6]. A normal Gaussian probability density distribution was assumed for each of the design parameters. It is also assumed that the selected design parameter varies discretely and regularly from its lowest to its highest value. For each such value of the design parameter, the deflection at a selected point on the beam cross-section is calculated. Since the Gaussian probability density function is continuous and smooth, fitting curves based on a finite number of deterministic solutions must be established. These functions will be used primarily in a semi-analytical approach and will also serve as a stochastic basis for the MCS to determine random moments, with the expected value of the structural response being the most important. Probabilistic moments will be defined in the form of curves (for SAM and SPT) or a set of points (for MCS), as a function of the coefficient of variation of the design parameter. Additionally, in a semi-analytical approach, one can easily define random moments as a function of two variables – the mean value and the standard deviation of the design parameter, and determine the so-called random response surfaces of the structure. Finally, by determining the so-called random entropy it will be possible to estimate the so-called measures of structural reliability expressed e.g. by the index First-Order Reliability Method (FORM).

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