Mathematical and Numerical Modelling of Materials and Structures with Functionally Variable Properties

Patryk Heród[1,*], Anna Jerzyńska[2], Szymon Hernik[2] and Halina Egner[2]

[1]Doctoral School, Cracow University of Technology, Kraków, Poland
e-mail: patryk.herod@doktorant.pk.edu.pl

[2]Faculty of Mechanical Engineering, Cracow University of Technology, Kraków, Poland
e-mails: anna.jerzynska@pk.edu.pl, szymon.hernik@pk.edu.pl, halina.egner@pk.edu.pl

KEYWORDS: *Functionally Graded Materials, Effective Properties, Composite Materials.*

Research on functionally graded materials (FGMs) is highly active, particularly in achieving desired property gradients for structural engineering applications. This field requires methods to: (1) determine effective FGM properties based on volume fractions and component configurations, and (2) optimize material profiles through strategic design of continuous variations in composition and microstructure to achieve location-specific optimal properties.

The aim of this research is to develop a methodology for designing the distribution of reinforcing particles in complex structural elements to achieve better performance parameters. The influence of the reinforcing particle distribution function on the distribution of fundamental mechanical quantities (stress, strain, displacement) in selected structural elements is analyzed using both analytical methods and finite element method (FEM). To estimate effective FGM properties, this research employs the energy equivalence hypothesis (Wiśniewska et al., 2019). Figure 1 presents exemplary results for a cantilever beam where the particle volume fraction $\xi(z)$ distribution is described by a polynomial function of the cross-section vertical coordinate, approximated in the FEM model by a stepwise change.

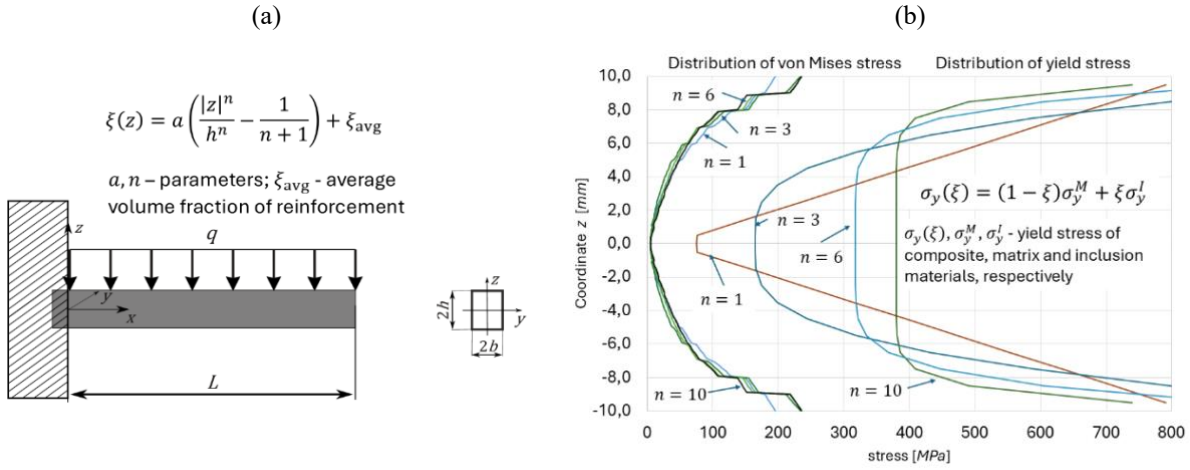

Figure 1. (a) Geometry and loading; (b) stress distribution in clamped cross-section.

The results obtained from the selected structural elements confirm that the applied method is capable of accurately determining the equivalent isotropic properties of composite materials required for the design of FGM structural elements.

[1] A. Wiśniewska, S. Hernik, A, Liber Kneć, H. Egner, *Compos B Eng.* **166**, 213–220, 2019. https://doi.org/10.1016/j.compositesb.2018.11.094