From inverse homogenization problems to closed-form models: simplified equivalent microstructure approach

Bartłomiej Bodak[1]*, Adrian Różański[1]

[1]Faculty of Civil Engineering, Wrocław University of Science and Technology, Wrocław, Poland
e-mail: bartlomiej.bodak@pwr.edu.pl

KEYWORDS: *micromechanics, effective properties, microstructure, electric permittivity.*

Predicting effective properties of random heterogeneous media remains an important challenge in micromechanics, particularly when phase property contrasts span wide ranges [1, 2]. Classical equivalent microstructure approach [3] provides a powerful framework for linking macroscopic response to simplified representations of complex media; however, it typically relies on solving an ill-posed inverse problem associated with significant computational cost.

In this work, we propose a generalized yet discrete formulation of the equivalent microstructure concept, with particular emphasis on its behavior across arbitrary phase contrasts. The proposed approach shifts the focus from detailed inverse reconstruction towards a reduced, forward modeling perspective, enabling a more direct characterization of the effective properties. At the same time, the key assumption of invariance with respect to the properties of the constituents is retained.

Building upon the Mori–Tanaka homogenization scheme [4], we introduce a minimal-parameter representation of the equivalent microstructure, in which combinations of a small number of discrete inclusion shape families are used to capture the effective response, thereby reducing the original continuous formulation to a low-dimensional, computationally efficient form. This formulation retains the key advantages of the original framework – namely its ability to represent a broad class of random media – while significantly reducing computational complexity and enabling partially closed-form expressions.

The approach is benchmarked in electric permittivity and thermal conductivity problems using several generated artificial and real microstructure representations. Performance of the proposed models is tested against the original model [3] and other semi-analytical approaches. The results demonstrate that the proposed formulation provides an accurate yet computationally efficient alternative for modeling effective properties across a wide range of phase property contrasts.

ACKNOWLEDGEMENT: The work was supported by the project *Minigrants* for doctoral students of the Wrocław University of Science and Technology.

[1] I. Sevostianov, M. Kachanov: Is the concept of "average shape" legitimate, for a mixture of inclusions of diverse shapes? Int. J. Solids Struct. 49, 3242–3254, 2012. https://doi.org/10.1016/j.ijsolstr.2012.06.018

[2] I. Sevostianov, M. Kachanov: On the possibility to represent effective properties of a material with inhomogeneities in terms of concentration parameters. Int. J. Solids Struct. 52, 197–204, 2015. https://doi.org/10.1016/j.ijsolstr.2014.10.003

[3] D. Łydżba, A. Różański, D. Stefaniuk: Equivalent microstructure problem: Mathematical formulation and numerical solution. Int. J. Eng. Sci. 123, 20–35, 2018. https://doi.org/10.1016/j.ijengsci.2017.11.007

[4] T. Mori, K. Tanaka: Average stress in matrix and average elastic energy of materials with misfitting inclusions. Acta Metallurgica. 21, 571–574, 1973. https://doi.org/10.1016/0001-6160(73)90064-3