# Macroscopic Yield Surfaces of Porous Solids with Anisotropic Matrix Behavior

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**KEYWORDS:** *Numerical limit analysis, Unit cell modelling, Porous plasticity, Anisotropic plasticity*

This study develops a theoretical and micromechanical framework to predict the macroscopic plastic response of porous materials with anisotropic matrix behavior. The formulation is built within the Gurson homogenization approach [1] and combines the Isotropic Plasticity Equivalent (IPE) concept of Karafillis and Boyce [2] with the procedure proposed by Benallal [3], enabling the development of macroscopic yield criteria for voided solids with general isotropic and anisotropic matrix constitutive laws. In the case of isotropic matrices, the framework accommodates general constitutive models, including Lode angle sensitivity. The approach naturally incorporates various material symmetries within the homogenization procedure, facilitating the derivation of anisotropic effective yield surfaces for porous solids.

For simplicity, the classical kinematically admissible velocity field proposed by Gurson is employed, although the methodology is general and allows for more refined trial velocity fields. Matrix anisotropy is represented via a traceless linear transformation within the IPE formulation. The macroscopic yield surface is obtained considering a spherical unit cell, yielding a parametric representation of the full six-dimensional effective yield surface. A numerical algorithm is developed to evaluate and visualize arbitrary sections of this surface, and illustrative results are provided for different matrix plastic behaviors, including Lode angle effects and distinct symmetry classes.

The analytical framework is validated against three-dimensional finite element simulations of hollow spherical unit cells with an isotropic or anisotropic matrix yield function, performed in Abaqus for a wide range of stress triaxialities, including both positive and negative values, and porosity levels. The proposed homogenization model shows very good agreement with the numerical predictions across all stress states, matrix responses, and porosity levels. The macroscopic yield locus is analyzed across the full range of triaxialities to illustrate its overall shape and anisotropy characteristics. In particular, at high positive and negative triaxialities, a comparative assessment of isotropic and anisotropic matrix behaviors is performed, highlighting the differences in the evolution and shape of the macroscopic yield surfaces.

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