# Comparative Study of Numerical and Analytical Solutions for an Elastic Half-Space under Rectangular Loading

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This work addresses the classical Schleicher-Love problem of an elastic, isotropic half-space subjected to a uniform vertical load applied over a rectangular area. While the theoretical foundation for such contact problems dates back to the Boussinesq solution, obtaining explicit analytical formulas for displacements throughout the 3D domain remains complex. This study utilizes the recent analytical solutions [1], which provide explicit formulas for the displacement components at any point in the half-space.

The primary objective of this research is to perform a comparison between analytical solutions and numerical results obtained through the Finite Element Method (FEM) using Abaqus. The study focuses on three key modeling aspects: the influence of mesh refinement on solution accuracy, a comparison between fixed boundary conditions and the use of infinite elements, and the evaluation of convergence behavior.

Preliminary results are presented in Fig.1 for a model utilizing infinite elements at the boundary with a progressively refined mesh, where the element size $h$ ranges from 1 to $\frac{1}{8}$ of the characteristic length. The results show good convergence in the energy norm, while the displacement field exhibits poor convergence. This occurs because the formulation with infinite elements does not fully constrain rigid body modes, leading to a constant shift in displacement field. The magnitude of this shift was observed to vary depending on the configuration of the infinite elements. Possible improvements to this problem will be discussed.

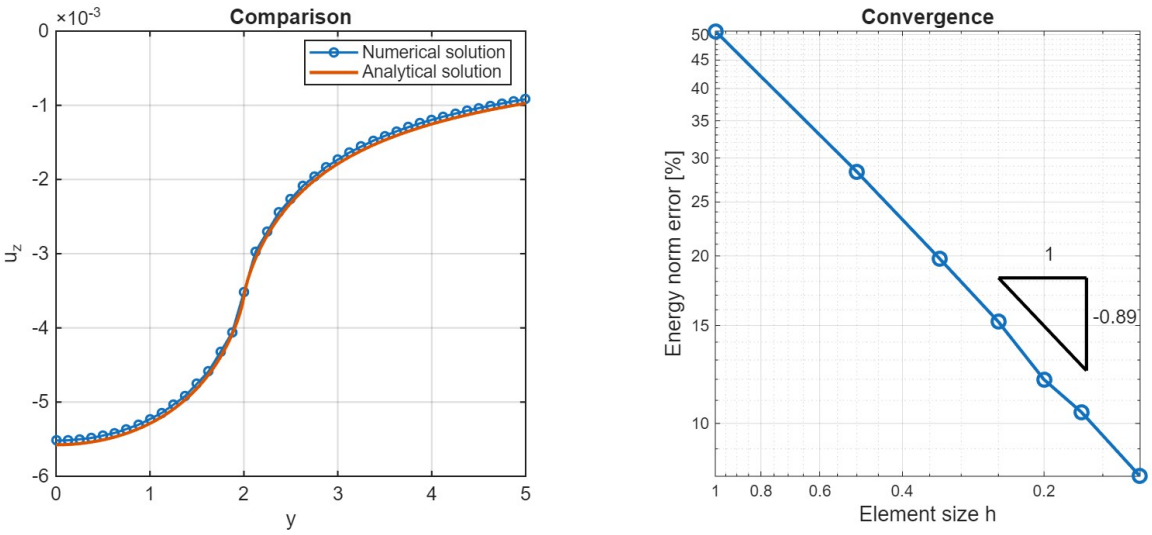


**Figure 1**: Left: comparison of vertical displacement $u_z$ along the $y$-axis between analytical and numerical solutions. Right: convergence in the energy norm measured in $5 \times 5 \times 5$ domain of the characteristic length of the rectangular load.

[1] M. Nagirniak, *ZAMM* **106**(1), e70310, 2026. https://doi.org/10.1002/zamm.70310