Epithelium and 2D Crystal

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KEYWORDS: *Lewis' Laws, Euler's Law, Dislocations, Cell Divisions, Energy, Tissue Stationary State.*

Epithelium is one of the most important tissues in the body, whether animal or plant. Externally, epithelial tissue presents a mosaic of densely packed polygonal cells. There are 4-, 5-, 6-, 7-, 8-, and 9-gons. Hexagonal cells constitute approximately 50% of the total, pentagonal and heptagonal cells approximately 45%, and the remaining 5% are tetragons, octagons, and 9-gons. A graph of the frequency of polygons versus the number of angles produces a curve that deviates slightly from the normal distribution. If only 5-, 6-, and 7-gons are considered, the proportions of polygons in the epithelium resemble those in an intermediate stage of hexatic transformation.

There is an analogy between epithelium and 2D crystal. The dense packing of cells in a single-layer epithelium and atoms in a 2D crystal, which is revealed by the honeycomb arrangement. More over, the lattice defects in both are similar. The difference is that in the 2D crystal, only pairs of 5- and 7-gons (5|7 dislocations) appear.

F.T. Lewis discovered laws of epithelial cell behaviour [1]. The first law states that the average number of cell sides is 6. Strictly speaking, this is a corollary of Euler's law for polygons. The second Lewis law states that the surface area S of a n-sided cell is proportional to the number $(n-2)$, i.e., $S = C(n-2)$, where C is a constant. The mathematical proof of this law uses Euler's polygon theorem and the principle of maximum entropy [2]. Therefore, it is also valid for 2D crystals. This allows us to estimate the relative changes in the lengths of the polygon sides. We have $a_5/a = 1.06$ and $a_7/a = 0.95$, where a_5 and a_7 are the lengths of the sides, and a is the length of the 6-gon side. Knowing these relative lengths and assuming the interaction potential of the crystal atoms, we can determine, for example, the dislocation energy.

Unlike a crystal, epithelial cells can undergo division. The mother cell loses its "identity" and its n sides are distributed between two daughter cells, and some of its neighbours gain new edges (because the new edge of the daughter cells touches them). Lewis' third law states that the average number of sides of dividing cells is 7. The question arises as to how the proportions of the polygons in the tissue are preserved. We consider the system of equations $\mathrm{d}f/\mathrm{d}t = \lambda g$, $\mathrm{d}g/\mathrm{d}t = \mu f$ (f – number of 5|7 pairs, g – number of 6-gons, t – time, λ and μ - constants) whose solutions, asymptotically in t, yield constant proportion $f/g = \sqrt{\lambda/\mu}$.

[1] F.T. Lewis, *The correlation between cell division and the shapes and sizes of prismatic cells in the epidermis of Cucumis*, The Anatomical Record, **38**, 341–376, 1928.

[2] N. Rivier, A. Lissowski, *On the correlation between sizes and shapes of cells in epithelial mosaics*, Journal of Physics A: Mathematical and General, **15**(3), L143, 1982.