A general self-consistent clustering analysis for computational homogenization using minimal offline stage

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Data-driven self-consistent clustering analysis (SCA) [1][2] has emerged as a robust reduced-order modeling framework for efficient computational homogenization of heterogeneous materials. However, its conventional offline stage remains computationally expensive, since it relies on multiple full-field direct numerical simulations (DNS) on finely discretized voxel-based representative volume elements (RVEs) to extract local strain concentration tensors. In addition, the resulting clustering patterns are inherently dependent on the elastic properties of the constituent phases, thereby limiting transferability and necessitating repeated re-clustering when material parameters are altered. In this work, inspired by the fundamental solution [3] for the static linear elastic equation together with the equivalence between polarization stress and eigenstrain [4], a uniform polarization-driven domain decomposition strategy is proposed to overcome these limitations. Six unit orthogonal polarization stress tensors are sequentially applied within each individual material phase, whereas zero polarization stress is imposed in the remaining phases. Induced strain fields inside each material phase are assembled into feature representations for unsupervised k-means clustering analysis [5]. The resulting clustering patterns depend solely on the underlying microstructural geometry and the Poisson ratio for a given choice of prescribed reference medium, thus ensuring enhanced generality and eliminating the need for phase-dependent recomputation. The proposed approach is systematically validated on a diverse set of representative microstructures, including periodic body-centered cubic (BCC) lattices (both percolating and non-percolating), as well as more complex random matrix-inclusion systems such as high-volume fraction Boolean–Poisson media and randomly distributed short-fiber composites. Effective elastic properties are computed by solving the Lippmann–Schwinger equation at the cluster level with a self-consistent scheme that involves iterative updates of the reference medium. The numerical results demonstrate that accurate predictions can be achieved with as few as 64 clusters per phase, even for complex phase distribution microstructures. The proposed method shows excellent agreement with high-fidelity FFT-based homogenization results, while significantly reducing the computational cost of the offline stage compared to the standard SCA framework. Furthermore, the proposed framework outperforms conventional mean-field approaches, as it remains accurate across the full range of volume fractions for RVE/RUC systems.

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