Sensitivity-based interval uncertainty analysis for free vibrations of variable stiffness plates with curvilinear wires

Magdalena Łasecka-Plura[1,*] **and Mieczysław Kuczma**[2]

[1]Faculty of Civil and Transport Engineering, Poznan University of Technology, Poznań, Poland
e-mail: magdalena.lasecka-plura@put.poznan.pl

[2]Faculty of Civil and Transport Engineering, Poznan University of Technology, Poznań, Poland
e-mail: mieczysław.kuczma@put.poznan.pl

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This work addresses the problem of free vibrations of thin variable-stiffness plates reinforced with discrete curvilinear wires modeled as one-dimensional inclusions embedded in a two-dimensional plate continuum. The aim is to formulate a numerical model and derive eigenvalue sensitivity measures with respect to parameters describing the wire layout, and then to employ these measures in a non-probabilistic uncertainty analysis. In contrast to classical laminate models, in which variations of wire orientation are incorporated in an averaged sense through local stiffness matrices of individual layers, the proposed approach treats the wires explicitly as discrete one-dimensional elements coupled with the plate kinematics [1].

The plate is discretized by the finite element method within the framework of the classical Kirchhoff theory of thin plates. The global stiffness and mass matrices of the system result from a superposition of the plate contribution and the contributions of individual wires embedded in its domain,

$$\mathbf{K}(\mathbf{p}) = \mathbf{K}_p + \sum_{i=1}^{N_w} \mathbf{K}_{w,i}(\mathbf{p}_i), \qquad \mathbf{M}(\mathbf{p}) = \mathbf{M}_p + \sum_{i=1}^{N_w} \mathbf{M}_{w,i}(\mathbf{p}_i) \tag{1}$$

where $\mathbf{K}_p$ and $\mathbf{M}_p$ denote the stiffness and mass matrices of the Kirchhoff plate, whereas $\mathbf{K}_{w,i}$ and $\mathbf{M}_{w,i}$ are the contributions associated with the i-th embedded wire. Such a formulation makes it possible to account directly for both the position of the wires and their curvilinear layout within the finite element model.

This leads to an eigenvalue problem in which the stiffness and mass matrices depend on the vector of parameters $\mathbf{p}$ describing the wire arrangement. Analytical sensitivity measures of the eigenvalues with respect to the design parameters are then determined, which makes it possible to quantify the influence of the wire layout on natural frequencies [2]. These sensitivity measures depend on three main factors: the variation of the wire geometry, the variation of the embedding operator, and the variation of the integration measure along the wire path.

The determination of sensitivity measures then provides the basis for taking parameter uncertainties into account, with the uncertain quantities treated as interval numbers. As a result, lower and upper bounds of the natural frequencies are obtained, which makes it possible to estimate the variability range of the structural response without exhaustive examination of all vertices of the uncertainty space [3].

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[1] S. Honda, Y. Narita, *J. Sound Vib.* **331**, 180–191, 2012. https://doi.org/10.1016/j.jsv.2011.08.019

[2] R.M. Lin, J.E. Mottershead, T.Y. Ng, *Mech. Syst. Signal Process.* **138**, 106536, 2020. https://doi.org/10.1016/j.ymssp.2019.106536

[3] A. Sofi, G. Muscolino, I. Elishakoff, *J. Sound Vib.* **347**, 79–95, 2015. https://doi.org/10.1016/j.jsv.2015.02.037