# Shape optimization of nonlinear elastic structures

**Przemysław Sobczak[1,*]**

[1]Faculty of Civil Engineering, Warsaw University of Technology, Warsaw, Poland
e-mail: przemyslaw.sobczak@pw.edu.pl

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A classical objective functional in structural optimization is compliance (the work of external loads on displacements). This functional depends on the geometry of the domain as well as on the solution of a boundary value problem of linear or nonlinear elasticity theory formulated on that domain. In such a framework it is convenient to work with the gradient-like method based on a concept of a shape derivative and level-set method. The key idea of level-set method is to represent the structural boundary with zero level-set of given function (level-set function – LSF). Now, changing the shape of a structure under optimization is equivalent to transport the LSF in such a direction that ensures decreasing the value of the objective functional. In this work volumetric form of shape derivative is used together with linear transport equation for evolution equation governing the change of shapes in a process. It is showed that it can be efficiently used to obtain optimal shapes in 2D elasticity shape optimization problems as well as in 3D case (see Fig. 1).

Moreover, influence of nonlinear elasticity of small displacement for shape optimization is studied. A nonlinear theory of elasticity for small displacements, incorporating a nonlinear (more realistic) constitutive relationship between the strain and stress tensors with sufficiently smooth potentials (convex and coercive), makes it possible to formulate structural shape optimization problems that can be solved using well-established mathematical tools, while at the same time addressing scenarios closer to reality (see Fig.2 for benchmark example).

Figure 1. Optimal shape for 3D case of a cantilever in linear elasticity.

Figure 2. Optimal shape for 2D case of a cantilever in nonlinear elasticity (left), first order optimality condition on boundary – constant value of function derived from solution of nonlinear elasticity and adjoint state (right).

## Literature:

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