On Range of Values of Cosserats' Material Parameters

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KEYWORDS: *Cosserats' Theory, Size Effect, Material Parameters, Pure Bending, Virtual Experiments.*

The early work of Gauthier and Jahsman [1] is among the first to expose a seemingly unexpected behaviour of granular materials whereby smaller specimens exhibit a softening, rather than stiffening behaviour predicted by the manner in which Cosserats' theory of elasticity has been postulated. Bigoni and Drugan [2] have later performed a detailed theoretical analysis to explain such a behaviour as in fact expected when the material inclusions are stiffer than the embedding matrix, which is exactly what they are in the tests performed in [1].

When this is regarded through the prism of the material parameters in Cosserats' elasticity, it turns out that such a behaviour disqualifies the theory from being a viable mechanical model as it necessarily implies that they fall outside of the range necessary for the strain-energy density of the material to remain positive. While numerical tests, such as e.g. those in [3], confirm that they do, there remains an unanswered question as to why such a mechanical problem remains stable indicating a positive accumulated strain energy upon application of a positive external work increment?

In our recent work [4] we have addressed this controversy by performing a detailed eigenanalysis of the problem and checked if the eigenvectors of the Cosserats' constitutive matrices may represent a deformed shape of the specimen/finite part thereof, i.e. if they may exist on a non-infinitesimal scale. If not, the fact that a strain-energy *density* may drop to zero at some point in a specimen does not necessarily imply a zero strain-energy *over a finite volume* (whole specimen or its representative volume element).

Taking pure bending of a unit strip of thickness h and Poisson's ratio n as a test case, it may be shown that the condition for a stable solution imposed on the characteristic bending length l_b is not $l_b > 0$ as would follow from the condition of positive strain-energy density, but in fact

$$l_b > -\frac{h}{24(1-n)}.$$

By pursuing the methodology presented in [5] it may be demonstrated that this is indeed the limiting value of the characteristic bending length that may be obtained in real periodic structures of the type analysed in [4] for any Poisson's ratio and any matrix-to-inclusion stiffness ratio.

ACKNOWLEDGEMENT: This research has been supported by the Croatian Science Foundation Research Project ANEMIC PM (HRZZ-IP-2024-05-9904).

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