Plates and shells reinforced with embedded shape memory alloy wires

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Plate and shell structures play a fundamental role in engineering applications. Shells exhibit a high stiffness-to-weight ratio and the ability to carry loads primarily through membrane action. In recent years, increasing attention has been devoted to the modeling of reinforced and smart structures. In particular, embedding one-dimensional reinforcements such as fibres, rebars, or wires into plate and shell structures provides a flexible means of tailoring their mechanical response. While continuous reinforcement is often modelled through homogenization, discrete reinforcements require a coupled multi-dimensional formulation. Shape memory alloys (SMAs) provide an additional level of functionality through their ability to undergo phase transformations and exhibit hysteresis (energy dissipation, damping, re-centring). Variational and energetic approaches to hysteresis modelling offer a mathematically consistent framework for describing such behaviour.

This work presents a variational formulation of thin plate and shell structures reinforced with discrete SMA wires. The plate/shell structure is modeled using Kirchhoff–Love kinematics, while the reinforcement is represented by embedded truss elements. The SMA behaviour is described within a rate-independent energetic framework consistent with hysteresis models ([1, 2]). The formulation naturally leads to a global–local finite element algorithm suitable for nonlinear analysis of adaptive plate and shell structures. The structure is discretized by the finite element method within the framework of the classical Kirchhoff-Love theory ([3]). The global stiffness of the structural system is obtained as a superposition of the plate/shell contribution and the contributions of individual wires embedded in its domain,

$$\mathbf{K}(\mathbf{z}) = \mathbf{K}_s + \sum_{i=1}^{N_w} \mathbf{K}_{w,i}(\mathbf{z}_i) \tag{1}$$

where $\mathbf{K}_s$ denotes the stiffness matrix of the plate/shell structure, and $\mathbf{K}_{w,i}$ are the contributions associated with the i-th embedded wire, that depend on a set of internal variables $\mathbf{z}$ describing the phase transformation process. This formulation makes it possible to account directly for both the spatial distribution of the wires and their evolving stress–strain–phase transformation state within the finite element model. The coupled evolutionary variational inequality is solved incrementally as a sequence of complementarity problems, allowing for an accurate representation of the path-dependent behaviour and the associated hysteresis loops.

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