Wave dispersion modeling for mechanical metastructures with nonlocal connections

Adam Martowicz[1,*] and Grzegorz Litak[2]

[1]AGH University of Krakow, Faculty of Mechanical Engineering and Robotics,
Department of Robotics and Mechatronics, Mickiewicza 30, 30-059 Kraków, Poland
e-mail: adam.martowicz@agh.edu.pl

[2]Lublin University of Technology, Faculty of Mechanical Engineering,
Department of Automation, Nadbystrzycka 36, 20-618 Lublin, Poland
e-mail: g.litak@pollub.pl

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The work addresses analytical and numerical investigation on the phenomenon of elastic wave propagation in mechanical metastructures, i.e., meso-scale periodic truss-like structures, that are formed using both local and nonlocal inner connections. Generally, metastructures are the systems in which the resultant behavior depends on the spatial distribution of the construction matter rather than on the material properties [1]. Considerable freedom regarding spatial shaping of metastructures, which is feasible mainly due to the capabilities of additive manufacturing technologies, leads to many interesting physical properties, that are of high importance in practical applications, e.g., quasi-zero stiffness in case of energy harvesters [2]. In the present research, the contribution of nonlocal links defined between the surrounding degrees of freedom results in an unique capability of forming dispersion curves.

The current work focuses on mathematical modelling and computational aspects for designing metastructures of the desired dynamic properties. The developed computational framework assumes definition of an unit cell for the considered periodic system, using finite element (FE) method. All the involved system connections are used to assemble the FE global stiffness and mass matrices, adequately applying the coordinate system transformations. The process of dispersion curves extraction is based on the solution of eigenvalue problem formulated with the Kelvin-Christoffel equation [3]. The relevant part of the resultant governing equation, that corresponding to the unit cell's central node, is used to find the above-mentioned curves. The applied homogenization allows for effective and reliable characterization of the system's dynamic response. The work is complemented with the results of numerical simulations conducted for the initial value problem.

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