Cobalt Nanorods Buckling within Nonlocal Eringen Elasticity Theory: Numerical and Experimental Approach

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KEYWORDS: *nonlocal elasticity theory; nanorods; post-buckling behavior.*

This study examines the post-buckling response of compressed nanorods using Eringen's nonlocal elasticity theory. We develop a nonlocal elastica model for a compressed nanorod based on the moment-curvature relation and the corresponding equilibrium equations. The boundary conditions are chosen to reflect those used in experiments on nanowires. The resulting nonlinear boundary-value problem is solved numerically with a shooting method, which makes it possible to compare the classical Euler elastica with its nonlocal extension and to evaluate the influence of the nonlocal parameter on the structural response. In particular, the simulations are used to observe how variations in the nonlocal parameter affect the post-buckling shape, as well as the value and location of the maximum deflection along the nanowire's axis. The shooting method was chosen because of its numerical stability and robustness when dealing with high order nonlinear differential equations containing trigonometric terms and variable coefficients. The effects of the scale parameter, slenderness ratio and boundary conditions on buckling behavior are very carefully examined.

The model was developed to match as closely as possible the conditions of the in-situ SEM compression experiments performed with a flat-punch nanoindenter. In those tests, SEM image sequences were used to track cobalt nanowire deflection in the post-buckling regime, providing the basis for fitting the nonlocal parameter in the model, for given values of the initial angle and bifurcation mode. The sensitivity of the elastica shape to variations in the nonlocal parameter is evident. The numerical results clearly show that even relatively small changes in the nonlocal parameter can lead to noticeable differences in the predicted post-buckling configuration of the nanorod, including the location and value of the maximum deflection. It can also be observed that the higher the value of the initial angle is, the greater the difference between the Euler-Bernoulli theory and the nonlocal Eringen theory. A similar conclusion can be drawn by considering the bifurcation mode value – the higher the mode value is, the greater the difference in the results. Likewise, higher buckling modes are associated with more pronounced discrepancies between the two models, suggesting that nonlocal effects become increasingly important for more complex deformation patterns.

On this basis, we conclude that selecting an appropriate value of the nonlocal parameter is not just a technical detail, but a key step in obtaining realistic predictions for a given material system. In particular, for larger initial imperfections, the nonlocal model departs more strongly from the classical Euler–Bernoulli prediction, which indicates that geometric imperfections amplify nonlocal effects. These studies showed overall agreement with elastica-based predictions, but also highlighted the influence of geometric parameters and the need to account for realistic support conditions at the nanoscale. Overall, the findings support future comparisons with nanoscale experiments and may help in identifying appropriate nonlocal parameter values and experimentally realistic boundary conditions for slender nanostructures.