On the Moving Interface Problem

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Numerous problems in material science and technology are related to varying interfaces. Let us mention such problems as crystalline grain growth or dissolution, austenite-martensite phase transformation, dislocation motion on slip plane, or frictional slip on contact surface. In the study of design improvement the interface shape is varied to obtain an improved or optimal design. In metal forming processes the interfaces between tool and work piece are modified to increase the product quality and reduce the work of acting loads.

The interface can be regarded as a surface on which static and kinematic fields suffer discontinuities. Several major types of interfaces can be considered. First, most popular is a coherent interface S^{i} specifying varying shape inclusion inducing stiffness increase, or growing damage zone with its interface boundary S^{d} to a limit state and ultimate failure. The discrete set of stiffeners in the form of thin shell or beam elements represent reinforcing interfaces S^{r}characterized by traction discontinuity. The softening response is generated by slip interfaces S^{sl} characterized by displacement discontinuity. Both elastic and dissipative slips then occur, such as friction, or dislocation slips.

On the interface S^{i} both displacements and surface tractions are continuous, thus $[u] = 0$, and $[T] = [\sigma n] = 0$, where u, ε, σ denote displacement, strain and stress fields, [ ] denotes the discontinuity of enclosed field symbol, thus $[u] = u_2 - u_1$. The displacement and traction gradients then are: $\left[u_{i,j}\right] = a_i n_j$ and $\left[T_{i,j}\right] = t_i n_j$.

The reinforcing interface S^{r} is represented by a thin shell inclusion of different stiffness from the matrix material or a thin layer of concentrated eigenstrains. This case can also correspond to a grain boundary or to an austenite-martensite interface. The traction discontinuity is related to interface generalized forces N through local equilibrium equations, thus: $[u] = 0$, and $[T] = [\sigma n] = L(N,q)$, where L is a differential operator and q characterizes geometric interface parameters.

On the softening or slip interface S^{sl}: the traction vector is continuous, but the displacement vector suffers discontinuity, so that: $[T] = 0$, $[u] = u^{\mathrm{s}} = B\,T$, where B is the symmetric compliance matrix. The interface can be conceived as a thin surface layer undergoing localized shear strain, such as that growing in a delaminated inclusion boundary zone. Usually, the friction slip accompanies elastic deformation, then friction slip rules should be introduced.

The evolution of interfaces can now be specified by introducing a transformation velocity field $\dot{\varphi}(x,t)$, such that each material element undergoes the shape transformation $x = x_0 + \dot{\varphi} dt$ with no stress introduced. The rate (or incremental) formulation is presented for 4 types of interfaces. The limit state conditions for moving damage interfaces and post critical states are examined. A hysteretic response on the frictional interface in reciprocal slip-sliding regime is discussed, presenting dissipated and stored energy balance.