# Incorporation of cross-anisotropic small strain stiffness into the Hardening Soil model

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The Hardening Soil (HS) model [5] is widely used as a standard simulation tool in advanced geotechnical analyses. It is implemented in many commercial finite element codes, and its parameters are typically derived from standard laboratory or field tests, or from empirical correlations. The model incorporates well-established concepts in soil mechanics, including barotropy, dilatancy, shear strength, and nonlinear pre-failure stiffness. In its basic form, the elastic component of the HS model is isotropic and hypoelastic. In a recent refinement of the HS model, a small-strain stiffness extension, referred to as HS-Brick, was introduced [3]. This extension accounts for the degradation of small-strain stiffness. It captures the initially high stiffness of soil before shear strains are mobilized, enabling more realistic predictions of deformation in engineering practice. It preserves the same general assumptions and material parameters as the widely used HSS model [1], but without the significant issue of stiffness overshooting observed in the HSS model. Both the original HS model and its small-strain extensions remain entirely isotropic. While accounting for the nonlinearity of small-strain stiffness represents a significant improvement, numerous studies have shown that incorporating anisotropy of small-strain stiffness leads to much more accurate deformation predictions when compared with geotechnical measurements. Despite its importance, anisotropic pre-failure stiffness is rarely implemented in commercial constitutive models due to the increased complexity of their formulation and calibration. Its practical application is also limited by the scarcity of reliable parameters describing anisotropy. However, recent advances in field and laboratory testing have enabled more precise investigations of anisotropic soil behavior. In most cases, cross-anisotropy is assumed to describe the directional dependence of soil mechanical properties. In elastoplastic modelling, cross-anisotropy can be introduced in both pre-failure stiffness and ultimate shear strength (e.g., via strength criteria or yield surfaces). The Anisotropic HS-Brick model (AHS-Brick), proposed and validated in this presentation, is the first extension of the HS-Brick model to incorporate inherent cross-anisotropy, limited to the pre-failure range [2]. Cross-anisotropy is introduced through a specialised scaling of the isotropic hypoelastic stiffness matrix [4]. The model is primarily intended for simulating the pre-failure mechanical behaviour of cross-anisotropic, stiff, fine-grained soils. Details of the finite element implementation are provided, along with selected element-level tests. Finally, the model performance is evaluated using an exemplary boundary value problem involving tunnelling-induced deformations. The results are analysed and discussed.

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