Quasi-static problems of the coupled linear theory of elastic nanomaterials with triple porosity

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Abstract

This paper considers the coupled linear quasi-static theory of elasticity for nanomaterials with triple porosity. The model combines Darcy's law with the concept of volume fractions corresponding to three levels of pore networks, namely macro-, meso- and micropores. The governing system of equations are formulated in terms of the displacement vector, the changes of pore volume fractions, and the fluid pressures in the three pore systems. By combining the constitutive relations, equilibrium equations, and fluid mass conservation laws, a general three-dimensional system of field equations is obtained for triple-porosity elastic nanomaterials.

Assuming harmonic time dependence, the governing equations are reduced to a system of partial differential equations with constant coefficients describing steady vibrations. An appropriate matrix differential operator is introduced, and the system is written in operator form for a nine-component vector function. Within this framework, the fundamental solution of the steady vibration system is constructed explicitly. The fundamental matrix is represented in terms of elementary functions, and its main properties are established, including the structure of singularities, behavior near the origin, and regularity away from the singular point.

The internal and external basic boundary value problems describing steady vibrations within this theory are formulated for regular vector functions, where the unknowns are the displacement vector field, the changes of pore volume fractions, and the fluid pressures in the three pore systems. Green's first identity is derived and used to prove uniqueness theorems for classical solutions. Green's second identity is obtained in operator form, and Green's third identity gives integral representations of solutions through surface and volume potentials constructed with the fundamental matrix.

In the analysis, surface potentials, both single-layer and double-layer, as well as volume potentials are introduced, and their basic properties are examined. A set of relevant singular integral operators is constructed, for which Noether-type theorems are shown to hold. The symbolic determinants and indices associated with these operators are explicitly computed. The boundary value problems are then transformed into equivalent systems of singular integral equations. Ultimately, by applying the potential method, the boundary integral equation approach, and the theory of singular integral equations, the existence of classical solutions to all formulated boundary value problems is rigorously established.

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