Initial stability of plates considering internal constraints by the Stochastic Boundary Element Method

Michał Guminiak[1,*], Marcin Kamiński[2]

[1]Faculty of Civil and Transport Engineering, Poznan University of Technology, Poznan, Poland
e-mail: michal.guminiak@put.poznan.pl

[2]Faculty of Civil Engineering, Architecture and Environmental Engineering, Lodz University of Technology, Lodz, Poland, e-mail: marcin.kaminski@p.lodz.pl

KEYWORDS: *Boundary Element Method, Thin Plates, Initial Stability, Semi-Analytical Probabilistic Analysis, Stochastic Perturbation Technique, Monte Carlo Simulations.*

The work presents the initial stability of plates in terms of the Stochastic Boundary Element Method (SBEM), what the authors discussed in [1] for plates supported only along its edges. It is assumed that the girders are loaded in their plane in a constant or linearly variable manner in one direction. The Boundary Element Method (BEM) is used to solve a single deterministic problem. The boundary and domain integral equations are formulated using e.g. Betti's theorem with fundamental solution applied to plate statics (Green's function). These equations are formulated in a singular approach and, alternatively, in a non-singular approach, where the collocation point is located outside the plate domain and at the same time sufficiently close to its edge. A plate boundary is divided into finite number of elements of the constant type. Internal constraints are considered too. The curvatures within the plate domain must be determined. For this purpose a plate domain is divided into finite number of sub-domains coupled with e.g. one central collocation point. This can be accomplished by using the first boundary and domain integral equation, which must first be differentiated twice with respect to a coordinate parallel to the applied load in the plate plane. This curvature can also be determined by simple difference quotients constructed for three internal collocation points located within a single subdomain – the central one and two others lying on one line before and after it (triple collocation point approach). Eliminating the boundary quantities from the governing equations leads to the standard eigenproblem, whose solution is the eigenvalues and eigenvectors that is the values of the critical multiplier and the corresponding curvature vectors. Alternatively, the second boundary and domain integral equation may be established by the addition second collocation point (double collocation point approach). Then there is no need to use the fundamental solution obtained for a unit moment whose vector is perpendicular to the normal to the plate edge.

The random approach requires a finite number of deterministic solutions obtained for different values of, for example, a single design parameter e.g. Young's modulus or a plate thickness. Based on this, a continuous polynomial response curve of the structure is determined, which will be integrally related to the probability density curve. For each of the design parameters the normal Gaussian probability density distribution is assumed. Three probabilistic approaches are applied: Semi-Analytical Method (SAM), Stochastic Perturbation Technique (SPT) e.g. [2], and Monte Carlo simulation (MCS). Probabilistic moments are defined in the form of curves (for SAM and SPT) or a set of points (for MCS), as a function of the coefficient of variation of the selected design parameter. The random approach allows testing the solution of the problem in the case of a non-singular approach with the use of double and triple collocation point technique.

[1] M. Guminiak, *Stability of rectangular Kirchhoff plates using the Stochastic Boundary Element Method*, Engineering Analysis with Boundary Elements, 144, 441–455, 2022

[2] M. Kamiński, *The Stochastic Perturbation Method for Computational Mechanics*, Wiley: Chichester; 2013