Soft Boundary Conditions in the Reissner–Mindlin Plate Analysis by the Equilibrium Finite Element Method

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In the equilibrium finite element approach to the analysis of the Reissner–Mindlin plate, bending and twisting moments, along with shear forces, are expressed in terms of first and second derivatives of the Southwell stress vector function [1]. The boundary conditions then take the form of linear constraints on the degrees of freedom at nodes located along the plate boundary.

The statically admissible solution of the Reissner–Mindlin plate, obtained using the Hsieh–Clough–Tocher (HCT) element presented in [1], yields excellent results for hard (2D) boundary conditions. Unfortunately, this approach suffers from over-constraining when soft (3D) boundary conditions are applied, as the number of required constraints exceeds the number of degrees of freedom associated with these conditions.

To overcome this difficulty, an additional term related to bending and twisting moments has been introduced in the present formulation. This modification makes it possible to impose independent constraints on the three stress resultants—namely, the bending moment M_n, the twisting moment M_{ns}, and the transverse shear force Q_n—defined with respect to the local coordinate system *ns*, associated with the plate boundary.

Two C^1 triangular elements were used in computations: the HCT element with 36 degrees of freedom and cubic interpolation functions, and the Argyris element with 63 degrees of freedom and fifth-degree shape functions. Elements with smooth shape functions are required because the shear forces, represented by second derivatives of the stress function, contribute to the strain energy.

The HCT element allows for the representation of an arbitrary parabolic distribution of bending and twisting moments along the edge and a linear distribution of the shearing force. The Argyris element enables the application of boundary conditions with bending and twisting moments following a quartic polynomial distribution and a cubic distribution of the shearing force.

The results obtained using the equilibrium FEM are dual to those derived from the kinematically admissible approach, which provides bounds on the exact value of the strain energy. The duality can be readily exploited to estimate the error of the approximate solution using Synge's metod [2].

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