Evaluation of Field Intensity Factors in 3D Potential and Elasticity Problems

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In material science and fracture mechanics, accurate evaluation of extremal, rather than average, values of physical fields is of prime significance. This requires studying high physical fields near edges of open surfaces, which are sources of unfavorable effects such as potential discharge, static fracture, fatigue, etc. Their strength is characterized by the field intensity factors (FIFs). The tail distributions of FIFs define the probability of rare high values, and serve to assign safety factors for a given fracture risk. To the date, FIFs are calculated mostly for 2D problems. In 3D, they are found for planar crack and for the axisymmetric spherical cap-like crack. Extensions to arbitrary open curvilinear surfaces have been hampered by the difficulties, involved by the need to solve 3D hypersingular boundary integral equation with the density, whose asymptotics defines the FIF. Nowadays, the *tangent-plane approach* (TPA) opens a possibility to remove the difficulties [1]. Our *objective* is to employ the new options for developing accurate and efficient method of FIF evaluation at edges of open surfaces in 3D. We reach the goal due to the improvements sketched below.

We consider an open surface, at which a physical field U is discontinuous. The edge contour is assumed to be a smooth curve. Then the field discontinuity ΔU near an edge point x has the monomial asymptotics

$$\Delta U(s) = K(x)s^{\alpha(x)} \tag{1}$$

where s is the distance to the edge point, measured in the cross-section normal to the contour; K is actually the FIF at this point; $\alpha(x)$ is the known exponent (usually, $\alpha = 1/2$). We account for the *asymptotics* (1) in approximation of the density ΔU. This significantly increases its accuracy at a vicinity of the edge.

Yet, the improved approximation of ΔU requires appropriate adaptation of the TPA to the monomial asymptotics (1). We performed the needed *modification* by complementing the TPA with the corrective terms derived. The modification drastically improves the accuracy of the density ΔU near the edge.

Furthermore, by (1), the FIF is defined as the limit of the ratio $K(x_j) = \lim_{s_j \to 0}\left[\Delta U(x(s_j))/{s_j}^{\alpha(x)}\right]$, in which both numerator and denominator go to zero. Consequently, with decreasing distance s_j, the relative error of conventional finding FIF via a single value of the pseudo-FIF $K_P(x_j) = \Delta U(x(s_j))/{s_j}^{\alpha(x)}$ unavoidably goes to infinity. To prevent the loss of the accuracy, we have designed *extrapolation formulae* of order $m > 0$, which employ $m + 1$ values of the pseudo-FIFs at $m + 1$ different distances from the edge. They provide notably better accuracy of FIFs, than the conventional calculations, corresponding to $m = 0$.

Summarizing, the objective to develop a universal method for accurate evaluation of FIFs in 3D potential and elasticity problems is reached by combining the three improvements briefly reported above. These are: (i) accounting for the *monomial asymptotics*; (ii) appropriate *modification* of the *TPA*, and (iii) *extrapolation formulae* of higher orders. Numerical results for a number of examples confirm that the method suggested provides accurate and computationally efficient evaluation of FIFs.

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