Numerical Anisotropy in Finite Elements: Where It Comes From and How to Eliminate It

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Accurate numerical modelling of wave propagation remains a challenge in computational mechanics, as spatial discretization may introduce artificial dispersion and anisotropy that are not present in the underlying physical system. Even for isotropic materials, finite element (FE) models may exhibit direction-dependent wave speeds and, more critically, frequency ranges in which wave propagation is completely suppressed. We adopted a broader definition of numerical anisotropy that includes not only directional variations of phase and group velocities, but also the presence of directional numerical frequency band gaps. From the perspective of wave modelling, such band gaps represent an extreme manifestation of anisotropy, as they completely eliminate wave propagation in parts of the angular spectrum.

The study focuses on three formulations: classical finite elements (FEM), time-domain spectral finite elements (TD-SFEM), and a class of spline-based finite elements (spFEM) developed by the authors [1]. Dispersion relations are obtained by combining the finite element formulation with Bloch–Floquet theory, allowing the analysis of the complete angular-frequency behaviour of the discretized system.

The results show that classical formulations of FEM and TD-SFEM exhibit pronounced numerical anisotropy, including directional band gaps which significantly limit their applicability in wave propagation problems. In contrast, the proposed spline-based formulation eliminates numerical frequency band gaps and substantially reduces directional dependence of wave velocities. As a consequence, a much larger portion of the numerical spectrum becomes usable, leading to a more efficient use of degrees of freedom.

Time-domain simulations of wave propagation confirm these observations. While standard formulations exhibit direction-dependent distortion and suppression of wave fronts, the spline-based model maintains stable and nearly isotropic propagation over a wide frequency range. Even near the upper limit of its applicability, the observed deviations remain small and systematic, without the abrupt breakdown associated with band-gap formation.

The presented results demonstrate that elimination of numerical band gaps is a key to improv the reliability and efficiency of finite element models for wave propagation. The proposed spline-based formulation provides a promising alternative to standard approaches, particularly in applications requiring accurate broadband simulations.

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[1] A. Żak and W. Waszkowiak. A spline-based FE approach to modelling of high frequency dynamics of 1-d structures. *Comput. Math. Appl.*, 2021, 104: 14-33.