MsFEM Supported by an Artificial Neural Network

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The Multiscale Finite Element Method (MsFEM), originally proposed three decades ago [1] and further developed in subsequent years, e.g., [2], is an efficient framework for multiscale analysis. In this approach, computations are performed at the macroscale while properly accounting for material properties and physical phenomena occurring at the microscale.

The core idea of MsFEM is the construction of special shape functions associated with coarse finite elements. These shape functions are computed offline by employing a fine-scale discretization within each coarse-mesh element and solving an auxiliary boundary value problem defined on the element domain. The resulting solution provides the degrees of freedom of the approximated shape functions, which constitute the columns of the prolongation matrix mapping degrees of freedom from the coarse grid to the fine grid. Through the Galerkin coarsening procedure, this prolongation matrix enables the computation of coarse-scale element matrices that incorporate the effects of heterogeneous material microstructures.

In certain applications, such as microstructure optimization [3] , it is necessary to compute the prolongation matrix multiple times for a single element. To enchance the efficiency of such analyses, we propose the use of artificial neural networks. The training inputs include material properties as well as arbitrary spatial distributions of components within the microstructure. Once trained, the neural network enables efficient evaluation of prolongation matrices without the need to repeatedly solve auxiliary boundary value problems. This approach is inspired by the application of artificial neural networks, known as Physics-Informed Neural Networks [4], to the direct (non-multiscale) solution of boundary value problems. Our preliminary results obtained for the Laplace problem with two materials distributed in a piecewise-constant manner confirm the potential of enhancing MsFEM through the use of artificial neural networks.

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