A Finite Element Model for Buckling Analysis of Composite Beams

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KEYWORDS: *Thin-Walled, Angle-Ply Laminate, Beam Model, Buckling.*

This study presents an enhanced stability analysis based on a shear-deformable beam formulation for laminated composite beam-like structures with thin-walled cross-sections. Each cross-sectional wall is modeled as a thin, symmetric angle-ply laminate. The governing equilibrium equations for a straight beam element are obtained using the principle of virtual work within a Lagrangian framework, incorporating Hooke's law and a nonlinear displacement field that accounts for restrained warping and large rotational effects in thin-walled sections.

The proposed formulation incorporates shear deformation effects through the Timoshenko–Ehrenfest theory to describe non-uniform bending behavior and the modified Vlasov theory to model non-uniform torsion. In addition, an advanced shear-deformable beam formulation is introduced to capture shear-induced coupling between bending–bending and bending–warping torsional responses. These coupling phenomena are particularly pronounced in beams with asymmetric cross-sections, where the principal bending axes do not align with the shear axes. The beam members are assumed to be straight and prismatic, and the applied external loads are considered static and conservative.

The element's geometric stiffness matrix is derived from a nonlinear displacement field of the cross-section, incorporating second-order displacement terms to represent large rotational effects. This formulation enables the evaluation of the incremental geometric potential associated with semitangential moments arising from internal bending and torsional actions, thereby ensuring equilibrium of moments at joints connecting beam members with differing spatial orientations.

A locking-free, super-convergent beam element is achieved by employing cubic interpolation functions for transverse displacements and twist rotations, together with mutually dependent quadratic interpolation functions for slopes and the warping parameter. This interpolation strategy accurately captures shear-deformable behavior without the need for reduced integration schemes to mitigate shear-locking issues.

To determine properties of a laminated thin-walled cross-section, a separate numerical model is developed. The cross-sectional properties are weighted by using three distinct reference moduli: the longitudinal modulus, which governs properties related to the coupling between normal stress forces and normal strains, including area, moments of inertia, area moments, sectorial inertial moments, and sectorial area moments; the shear modulus, which governs the torsional moment of inertia and shear coefficients; and the coupling modulus, which governs the coupling between the normal and shear strains. All cross-sectional properties are defined with reference to the midline of each branch of the cross-section.

ACKNOWLEDGEMENT: This work was supported by the Croatian Science Foundation under the project number [HRZZ-IP-2024-05-6868]. This work was Funded by the European Union – NextGenerationEU under project numbers (uniri-mz-25-3; uniri-iz-25-95 and uniri-iz-25-233).