Total and Modelling Error Estimation for the 3D-Based Hierarchical First-Order Shell Models

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In the case of thin-walled structures, error-controlled adaptive finite element (FE) analysis requires estimation of the total, approximation and modelling errors, in contrast to 3D-elasticity where only approximation error is required. The approximation and modelling error estimators control the *p*- and *q*-adaptivity in the longitudinal and transverse directions of the shell (or plate) structures, respectively. Here, *p* and *q* represent longitudinal and transverse approximation orders within adaptive shell finite elements. In this work, the order *q* characterizes also the order ($I \equiv q$) of the 3D-based hierarchical shell models M*I*. These 3D-based models apply through-thickness shell degrees of freedom (dofs), instead of mid-surface dofs of the conventional hierarchical shell models.

The applied equilibrated residual method (ERM) of approximation error estimation have been presented in [1] and [2] for the general elliptic and specific 3D-elasticity problems, respectively. The ERM approximation error estimation for the conventional and 3D-based hierarchical ($q \geq 1$) shell models M*I* ($I \equiv q$) has been proposed in [3] and [4], respectively. Application of the last two approaches to the total and modelling error estimation is possible for higher-order and first-order models ($q \geq 1$). However, the first-order hierarchical model ($I \equiv q = 1$) suffers from the improper solution limit and has to be replaced with the first-order ($q = 1$) Reissner-Mindlin (RM) model of the correct solution limit. Unfortunately, direct application of ERM to the total and modelling error estimation for this model does not lead to the correct estimation results as this model is not a regular member of the hierarchy of models M*I*. In this work, we show how to resolve the problem of incorrect total and modelling error estimation results for the RM model. Note that the approximation error estimation for this model is presented in [5].

One of the elements of ERM are local (element) problems, the solution of which leads to element error indicators. The removal of the incorrect estimation of the total and modelling errors for the RM model requires modification of these local problem through energy projection of the work of the RM-based inter-element forces of the element so as the local problem solution moves from the incorrect solution space to the projected solution space closer energetically to the M*I* solution space. The projection can be applied to Neumann and Dirichlet (with 6 rigid body vertex displacement constraints) genuine local problems or tuned both Neumann and Dirichlet problems with 18 vertex displacement constraints. We also show that the first-order hierarchical model M*I* ($I \equiv q = 1$) requires energy projection of the 18 vertex constraints in the Neumann and Dirichlet problems, so as the local problem solution moves from the incorrect solution space to the M*I* ($I \equiv q \geq 2$) correct space. Error estimation effectivity indices are used to demonstrate effectiveness of the proposed approaches.

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