Fast Numerical Method Based on Discrete Sine and Cosine Transforms for Wave Propagation in Heterogeneous Media

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FFT-based numerical methods, introduced by Moulinec and Suquet [1], are now a class of highly efficient solvers in computational micromechanics. Their success relies on their simplicity of implementation and their low computational cost, of order $O(N \log N)$, which is significantly lower than that of approaches based on the assembly and inversion of global matrices. However, the representation of fields using Fourier series inherently implies periodic boundary conditions. This limits their application to problems where more realistic boundary conditions are required, such as non-periodic microstructures or transient problems. Extending these methods to non-periodic frameworks is therefore a major scientific challenge.

Recent works [2] have extended these methods [1] to non-periodic boundary conditions using a decomposition of the solution field into a term satisfying the boundary conditions and a second term, referred to as a fluctuation, described by sine and cosine series and vanishing at the boundaries. This approach allows one to move beyond the strictly periodic framework and has enabled the solution of elasticity problems [3] for microstructures subjected to Dirichlet and Neumann boundary conditions.

In this work, this framework [2] is used to develop solvers for mechanical problems involving microstructures subjected to non-periodic boundary conditions. Starting from the classical elasticity formulation [3], an implicit method is proposed for elastodynamic problems with stress wave propagation in heterogeneous me dia, based on the approach developed in [2] and on a Newmark-β time discretization, following [4]. The method is validated using analytical solutions in both homogeneous and heterogeneous cases. It is then applied to study wave propagation in polycrystalline media with different levels of anisotropy and compared with a reference finite element solution.

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