Numerical Verification of Euler's Observation that Loading by Conservative Body Forces is Equivalent to the Pressure Reaction for the Incompressible Elasticity

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KEYWORDS: *Incompressible Elasticity, Conservative Body Force, Finite Element.*

We verify the hypothesis attributed to L. Euler [1] that loading by conservative body forces in finite incompressible elasticity is manifested only by a change of the pressure reaction without affecting the deformation of the body. The modification of the pressure reaction is by the negative potential provided that also the Neumann boundary condition is modified by the corresponding negative potential.

We experiment with the so-called Eriksen's class of controllable deformation [2] in isotropic incompressible elasticity. These are the problems for which exclusively stress loading on the boundary is prescribed and whose analytical solutions are found. The numerical solutions corresponding to the additional conservative body forces are obtained using the Simo, Taylor and Pister [3] finite element method for nearly incompressible elasticity with hexahedral elements. The technique is based on a mixed formulation using the displacements and the pressure. Such mixed formulation prevents the solutions from instabilities, oscillations and locking usually appearing when one approaches to incompressible material.

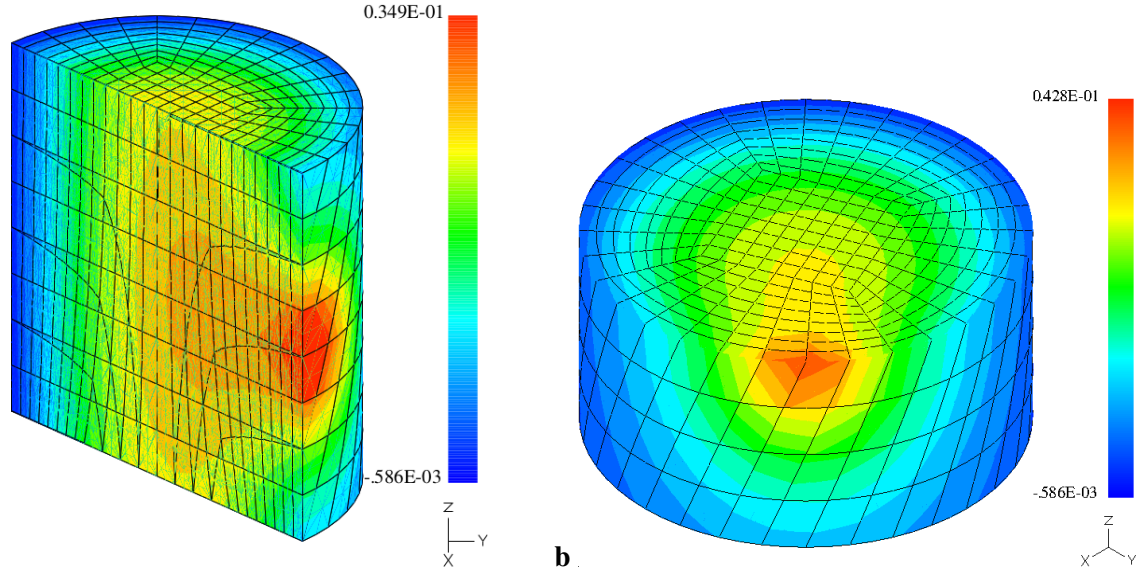

Figure 1: Pure torsion of a cylinder, $r = 1$, by angle of 45^0. Gravitational potential $\bar{v} = -0.02/r$, $r = \| x - x_c \|$, $x_c = (1.4/\sqrt{2}, 1.4/\sqrt{2}, 1)$: a) pressure over vertical, and b) horizontal cross-section of the cylinder (deformed configuration)

[1] C. Trusdell, R. Toupin, The Classical Field Theories, Springer, Berlin, Heidelberg, 1960, 226-858. https://doi.org/10.1007/978-3-642-45943-6_2

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