Application of the Pontryagin's Maximum Principle in shape optimization of Glue Laminated Timber beams and its numerical and experimental validation.

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The problem of shape optimization of a Glue Laminated Timber (GLT) beam of variable height under 4-point bending is posed and solved with the use of the Pontryagin's Maximum Principle (PMP) [1]. For a given reference beam of constant cross-section two optimization problems are stated: 1) minimization of an amount of material used (beam's volume) while preserving the original load carrying capacity (V_{min} beams) and 2) maximization the beam's load carrying capacity (LCC) while preserving the beam's volume (Q_{max} beams).

Governing equations for a beam of variable height were derived from the 1D Timoshenko beam theory and general equilibrium equations for a plane elasticity problem, accounting for boundary conditions on both top (flat) and bottom (oblique) faces of the beam. Author's energy-based limit state condition for anisotropic solids with asymmetric elastic range was used as a measure of material's effort [2]. Values of mechanical parameters of the material were assumed in the design process according to the EN 14080:201 standard. Control theory problem was formulated and solved with the use of the PMP, using the Dircol software which utilizes the direct collocation method. Obtained results were compared with the 3D Finite Element Analysis (FEA) results obtained from the Abaqus CAE software. Theoretical volume savings were approximately equal to 11,4% of the reference volume, while increase of the LCC was ca. 12,2% of the reference LCC.

After appropriate adjustments of the numerically determined beam's optimal shape, nine GLT beams were ordered, 3 of each type (reference beams, V_{min} beams and Q_{max} beams) and examined in the 4-point bending tests. Beam's deflections in nine points as well as the plane strain state near the bottom face in the midspan were recorded with the use of inductive displacement transducers and strain gauge rosette, respectively. Experimental finding only partially confirmed the results of numerical optimization due to large variation of the mechanical properties of GLT. Points of material's failure clearly corresponded with the designed points of maximum material effort.

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[2] P. Szeptyński, *Arch. Metall. Mater.*, **62**, 729-736, 2017. https://doi.org/10.1515/amm-2017-0110