Weak and Strong Form Multipoint Meshless Method for Elliptic Problems

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KEYWORDS: *Meshless Methods, Multipoint Method, MLPG, Weak Formulation, Strong Formulation, PINN*

While the Finite Element Method (FEM) remains the dominant approach for engineering problems, meshless methods offer attractive alternatives. A wide range of meshless methods differ from each other in the process of construction of local approximation. In general, they can be classified into methods based on strong formulation and methods based on weak formulations [1].

One of the basic meshless methods, the Meshless Finite Difference Method (MFDM), is typically described in strong collocation form [2]. However, recent developments extended this method, including its higher-order multipoint variant [3], to various weak formulations [4]: global variational (Galerkin, Petrov-Galerkin), energy functional minimization, and meshless global-local Petrov-Galerkin (MLPG), particularly the computationally efficient MLPG5 approach using Heaviside test functions.

The multipoint meshless FDM raises approximation order by introducing additional degrees of freedom at stencil nodes via moving weighted least squares (MWLS), enabling analysis of elliptic boundary value problems in both strong local and various weak global and global-local formulations without mesh densification. The method also shows potential for integration with physics-informed neural networks (PINNs) via MFDM discretizations in the PDE residual loss [5].

Numerical tests for 2D elliptic Poisson problems, plate bending, and prismatic bar twisting confirm that all weak formulations yield results comparable to strong form solutions while global-local MLPG5 form [6] significantly reduces computational effort through boundary-only integration.

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