Master Stability Function for networks of coupled piecewise-smooth oscillators

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The Master Stability Function (MSF) [1] has become a standard tool for analyzing the stability of complete synchronization in networks of identical oscillators. This framework offers two main advantages. First, it reduces the stability analysis of a high-dimensional network to the study of a low-dimensional variational equation governing perturbations transverse to the synchronization manifold. Second, the results obtained for a given oscillator model and coupling scheme can be readily extended to networks with different connection topologies. However, applying this framework to piecewise-smooth systems is significantly more challenging, as the variational equation is not defined at points where the vector field is non-differentiable.

To overcome this limitation and enable MSF analysis of piecewise-smooth systems, a novel extension of the MSF framework, based on a previously developed Jacobian estimation technique [2] is proposed. The central idea is to approximate the Jacobian matrix along a system trajectory using the evolution of small orthogonal perturbations. This enables the estimation of perturbation dynamics even when the trajectory passes through non-differentiable points, thereby allowing the computation of the MSF in the absence of a well-defined variational equation. A key advantage of the proposed framework is its straightforward applicability to a broad class of piecewise-smooth systems without requiring system-specific analytical derivations, while still providing quantitative insight into synchronization stability.

The proposed framework is presented and evaluated against existing approaches. Its applicability is demonstrated through the computation of Master Stability Functions for linear and nonlinear impact oscillators, while its accuracy is verified through comparison with an alternative MSF estimation method.

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[1] L. M. Pecora i T. L. Carroll, „Master stability functions for synchronized coupled systems," vol. 80, nr 10, p. 2109, 1998.

[2] M. Balcerzak, A. Dabrowski, B. Blazejczyk-Okolewska i A. Stefanski, „Determining Lyapunov exponents of non-smooth systems: Perturbation vectors approach," *Mechanical Systems and Signal Processing,* vol. 141, p. 106734, 2020.