Collocation-Based Robust Variational Physics-Informed Neural Networks (CRVPINNs)

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Physics-informed Neural Networks (PINNs) have been introduced by G. Karniadakis [1] as a way to apply Deep Learning methods to solving partial differential equations (PDEs). Unlike the more common data-driven approaches, PINNs operate by minimizing a loss function based on the residual of the strong formulation of the PDE, evaluated on a set of collocation points. Because of that, PINNs fail to provide accurate solutions in some problems with low regularity of the data, since the solution only makes sense in a variational form. A natural way to overcome this limitation is to build the loss function using the residual of the variational formulation of the underlying PDE – an approach known as Variational PINN (VPINN) [2].

While the VPINNs have been successfully applied in a number of areas, they have a considerable drawback in that the loss is sensible to the choice of basis functions, and may not be robust, that is, the loss value can tend to zero, while tue true error in a relevant Sobolev norm does not. To mitigate that problem, Robust Variational PINN (RVPINN) has been proposed [3], which follows the core ideas introduced in Minimum Residual (MinRes) methods. The RVPINN loss function is constructed as the dual norm of the residual of the variational (Petrov-Galerkin) form of a PDE. As long as the variational formulation satisfies the assumptions of the Babuška theorem (continuity and inf-sup stability of the bilinear form), such loss function constitutes an efficient and reliable estimator of the true error. More precisely, the norm of the true error is bounded from below and above (up to an oscillation term) by the square of the loss.

Unfortunately, to compute the weak residuals, RVPINNs require expensive numerical integration. We proposed an alternative combining their robustness with the efficiency of standard PINN – Collocation-based Robust Variational PINN (RVPINN) [3]. The continuous spatial domain and integral forms are replaced with a discrete set of collocation points and discrete weak formulation, mimicking the properties of the continuous, Sobolev space-based theory, similar to the finite difference method. The result is a robust loss function, which does not require integration.

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