Anisotropic elastic properties of smoothed interphase in heterogeneous materials

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This work addresses the description of the spatial distribution of mechanical fields for elastic heterogeneous materials by using the fast Fourier transform (FFT) numerical method [1]. It relies on the resolution of the elastostatic boundary value problem with the Green functions method. By using a classical approach in micromechanics, the original elastic heterogeneous media is replaced by an homogeneous elastic media with a heterogeneous polarization field to be determined. This leads to a Lippmann-Schwinger equation which can be efficiently solved by a FFT-based iterative scheme on a regular grid. This approach has been the subject of numerous developments and is now widely used in micromechanics [2]. However, this reformulation of the local problem, based on Fourier series, can lead to spurious oscillations on local fields. This phenomenon emerges when (i) the polarization field has some discontinuities and (ii) pseudo-spectral differentiation is used [3]. To tackle this problem, modified Green operators and interface spreading approaches have been proposed. The latter consist in the introduction of an interphase, whose properties have to be defined, between domains with different elastic properties. This approach can be performed locally using composite voxel techniques [4], or globally using smoothing techniques [3]. By reducing material discontinuities, it improves consistency with the Fourier-series representation of the local fields.

Building on the previous work of Morin *et al.* [3], the present contribution aims at presenting two improvements: first, to provide a smoothing of elastic properties which is invariant when applied to stiffness or compliances, and second, to consider general anisotropic elastic properties. To do so, use is made of the logarithm of elasticity tensors following ideas of Moakher and Norris [5] to define an invariant distance between tensors. Numerical experiments are performed on elasticity problems involving isotropic and anisotropic phases, for composites and polycristals. A significant reduction of spurious oscillations is observed on the local fields.

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