Multistable Shells Design for Energy Harvesting from Mechanical Vibrations

Lukasz Kloda[1,*]

[1]Department of Applied Mechanics, Lublin University of Technology, Lublin, Poland
e-mail: l.kloda@pollub.pl

KEYWORDS: *Geometric Nonlinearities, Multistable Structures, Nonlinear Dynamics, Energy Harvesting.*

Growing demand for self-powered sensors and autonomous monitoring systems has increased the need for robust vibration energy harvesters capable of operating under broadband and variable excitation (in both frequency and amplitude). However, classical linear resonant harvesters, such as cantilever beams or plates, suffer from narrow bandwidth and high sensitivity to frequency detuning, limiting their effectiveness in real-world conditions. To address these limitations, nonlinear approaches can be exploited, including nonlinear resonances, parameter tuning, and multistable designs. In this context, orthotropic multistable shells proposed in [1], characterized by one-sided clamped boundary conditions are promising. The shell mid-surface is defined in the (x_1, x_2, x_3) domain using two pseudo-radii $r_0(x_1 = 0)$ and $r_f(x_1 = L_1)$, see Fig. 1 and Eq. (1).

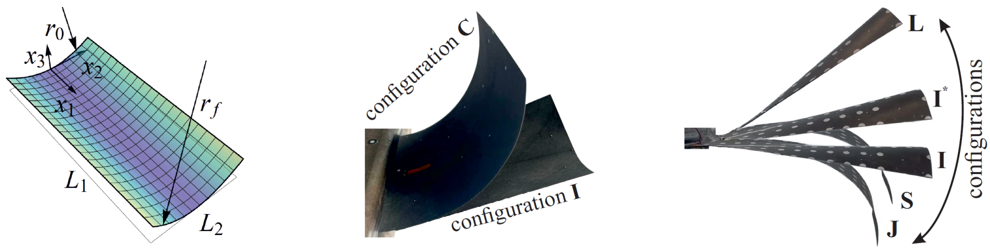

Figure 1. The shell geometry definitions (left), stable configurations for prototype **A** (center), and prototype **B** (right).

$$w_0(x_1, x_2) = \left[\frac{1}{r_0} + \left(\frac{1}{r_f} - \frac{1}{r_0}\right)\frac{x_1}{L_1}\right]\frac{x_2^2}{2}, \quad 0 \le x_1 \le L_1, \quad -\frac{1}{2}L_2 \le x_2 \le \frac{1}{2}L_2. \tag{1}$$

Energy harvesting from an MFC patch embedded in a bistable shell of type **A** ($r_0 = 0.1045$ m, $r_f = 0.08493$ m, $L_1 = 0.18$ m, $L_2 = 0.15$ m) is experimentally investigated [2]. The structure exhibit two distinct configurations **I** and **C** with significantly different natural frequencies in the linear regime. For moderate response amplitudes, the system shows nearly linear behavior in configuration **C** and pronounced softening behavior in the configuration **I** during in-well oscillations. At higher amplitudes, reversible snap-through transitions occur, significantly enhancing energy harvesting performance. In the second case, nonlinear dynamics and energy harvesting performance of a pentastable shell of type **B** ($r_0 = 0.114$ m, $r_f = -0.07$ m, $L_1 = 0.45$ m, $L_2 = 0.15$ m) are presented [3]. The latter sheds the light on effective placement of a piezoelectric transducer location based on criteria: stress distribution, strain distribution, frequency bandwidth, excitation framework and MFC patch integration in the structure. The results are summarized in maps of RMS power versus excitation frequency, highlighting the advantages of cross-well oscillations. A reduced-order model capturing both local and global dynamics of the multistable structure, including the piezoelectric coupling, is currently under development.

ACKNOWLEDGEMENT: This research was funded in whole or in part by National Science Centre, Poland 2021/41/B/ST8/03190.

[1] M. Brunetti, A. Vincenti, S. Vidoli, *Int. J. Solids Struct* **194**, 388–397, 2018. https://doi.org/10.1016/j.compstruct.2018.04.009

[2] A. Mitura, M. Brunetti, L. Kloda, F. Romeo, J. Warminski, *Mech. Syst. Signal Process* **206**, 110890, 2024. https://doi.org/10.1016/j.ymssp.2023.110890

[3] L. Kloda, *Mech. Syst. Signal Process* **244**, 113825, 2026. https://doi.org/10.1016/j.ymssp.2025.113825