A new method for solving 3D problems of out-of-plane or out-of-surface crack perturbations, with some applications to mixed-mode propagation of cracks

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KEYWORDS: *3D elastic body, arbitrary crack, out-of-plane perturbation, extended Bueckner-Rice theory, mixed-mode propagation.*

The aim of this work is to propose a new, simple and efficient method for solving problems of 3D elastic bodies containing cracks slightly perturbed *out of their original plane or surface*. It is divided into three parts.

We begin, in a first part, by defining a suitable extension of Bueckner-Rice's theory of 3D weight functions. Rice (1985, 1989)'s re-formulation of Bueckner (1987)'s theory provided the first-order variation of the elastic fields (displacements, strains, stresses) arising from an *in-plane or tangential* perturbation of the crack front, for an arbitrary crack in an arbitrary body. This result is extended here to arbitrary geometric perturbations of the crack front *and* surface, including an *out-of-plane or normal* component to the crack surface. The basis of the treatment is a new, general formula providing the variation of the total energy of the body arising from such an arbitrary crack perturbation. This formula is obtained by adapting and extending reasonings and results of deLorenzi (1982) and Destuynder et al. (1983). It is then used to derive the first-order expression of the full displacement field everywhere in the body. The reasoning here basically follows and extends that in the works of Rice (1985, 1989), which was limited to in-plane or tangential perturbations of the crack front.

In a second part, we illustrate the possible use of the formalism thus defined for the treatment of elastic problems of out-of-plane or out-of-surface crack perturbations. This is done by considering the simplest possible case of out-of-plane perturbation of a semi-infinite plane crack embedded in some infinite body. This problem was solved by Movchan et al. (1998), using an elaborate method specific of the special, infinite geometry considered. In constrast, the method of solution proposed here is general and potentially applicable to *any* cracked geometry. The derivation involves two steps :

1) In the general formula providing the variation of the displacement at any point of the body, we let this point of observation go to an arbitrary point on the crack surface, so as to get the variation of the displacement discontinuity there.
2) In the formula thus obtained, we let the point of observation on the crack surface go to some arbitrary point on the crack front, so as to get the variations of the stress intensity factors there.

The results obtained in this way fully confirm, and somewhat extend, those derived by Movchan et al. (1998) using a more complex and specific method.

We finally expound, in a third part, the possible applications of these results to the theoretical prediction of crack paths in 3D bodies loaded in mixed-mode conditions. We especially focus on the interpretation of the experimentally well-documented out-of-plane instability of crack fronts loaded in mode I+III. Three cases are considered, in order of increasing complexity :

1) That of a mode I+III loading, with a critical energy-release-rate G_c independent of mode-mixity.
2) Again that of a mode I+III loading, but with a mode-mixity-dependent G_c.
3) That of a general mixed-mode I+II+III loading, with a mode-mixity-dependent G_c.

The results suggest interesting interpretations of both old and recent experiments.